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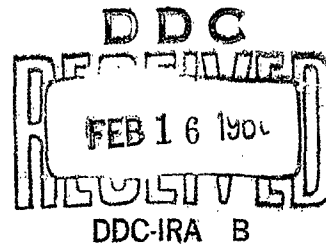
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BALLISTIC RESEARCH LABORATORY



REPORT



- ⑥ THE PROBABILITY OF HITTING VARIOUS PARTS OF AN AIRPLANE
AS DEPENDENT ON THE FRAGMENTATION CHARACTERISTICS OF
THE PROJECTILE,

BY

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THE PROBABILITY OF HITTING VARIOUS PARTS OF AN AIRPLANE
AS DEPENDENT ON THE FRAGMENTATION CHARACTERISTICS OF
THE PROJECTILE

Abstract

Methods of rating A.A. projectiles with regard to fragmentation efficiency are discussed. An expression for the probability of hitting a small projected area of an airplane is derived and also an expression for the probability of hitting the fuselage or wings. From these it appears that the fragmentation efficiency varies as the product of the number of effective fragments and their effective range. It is shown that with the present fire control equipment, the probability of a direct hit at high altitudes is as great as the probability of hitting the pilot. This result suggests the use of supersensitive fuzes instead of time fuzes for arming A.A. shell, even those of 3 $\frac{1}{2}$ caliber and larger.

* in

In the Ballistic Laboratory Report No. 125 entitled, "A Comparison of Antiaircraft Guns of Various Calibers", it was assumed that the ratio of the effectiveness of two shell is the mean of the ratios of their weights and the numbers of their fragments. It was pointed out in the report that this assumption was doubtful and was made because of inadequate knowledge. It is the purpose of this report to deduce expressions for the probability of hitting various parts of an airplane for the purpose of throwing additional light on the efficiency of projectiles as dependent upon their caliber and fragmentation.

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In Report No. 127, 'The Probability of Hitting an Airplane as Dependent upon Errors in the Height Finder and Director', it was considered that a 'hit' is obtained when the burst is contained within a volume of dimensions a, b, c, enclosing the airplane. This assumption was in accordance with the current procedure in antiaircraft target practices in the U.S. Army. According to this a burst is rated as a 'hit' if the burst is in such a position with respect to the center of the airplane as to produce at the airplane a fragment density, ρ , greater than .16 fragments per sq. yd.* It is obvious that for rating the general efficiency of antiaircraft fire, the mere evaluation of the probability of a 'hit' as defined above would be inadequate since (1) this procedure involves a discontinuity in the probability of hitting at the surface corresponding to $\rho = .16$ (2) considers the airplane as a point and (3) makes no estimate of the probable damage to the pilot or airplane.

We proceed to outline a method for evaluating the probability of disabling an airplane. The first step is to determine the probability of hitting various parts of the airplane; the second step is to determine from engineering or other considerations the parts of the airplane a fracture of which will cause the structural or other failure of the airplane. For example if the bomber is hit, the airplane will fail as a bomber. If the gas tank is perforated, a leakage of gas will occur. If certain members of the framework of the fuselage or wings are broken by a fragment, the fuselage will break in two. In the following we shall discuss in detail the probability of hitting various parts of the airplane but shall mention only incidentally the effectiveness of a hit by a fragment in disabling the airplane.

Probability of hitting a small projected area

We first consider the probability of hitting a small projected area, that is, an area projected on a surface normal to the trajectories of the fragments. Such a projected area might represent the pilot or the bomber. It is assumed that the dimensions of the area are small compared with the dimensions of the effective side spray so that

* There is no authoritative source for the belief apparently widely entertained, that a fragment density of .16 will necessarily disable the airplane.

except at points very close to the burst, the area can be placed within the side spray.

We define certain symbols representing the characteristics of the fragmentation and target etc.

Symbol	Definition	Unit	Remarks
A	= Projected area of target	yd ²	
r	= Distance of area from point of burst	yd	
N	= Number of effective fragments	-	This is considered to be a function of r.
Ω	= Solid angle of the side spray	radians ²	The fragments are considered to be distributed at random in this solid angle. The nose spray is neglected.

In Ballistic Research Laboratory Report No. 127, it is shown that the probability of placing a burst within a box of dimensions and volume a, b, c in the neighborhood of the airplane is

$$\frac{.0195 a b c}{a_h a_d^2 S R^3 t^2 \omega_\epsilon \omega_\alpha \cos \epsilon_f} *$$

However, as was mentioned on page 3 of Report 127, this expression was derived on the assumption that the time of flight, t, is correctly given. On account of errors

* In this

- S = present slant range
- R = future slant range
- t = time of flight
- ω_ε = angular velocity of airplane in elevation
- ω_α = angular velocity of airplane in azimuth
- ε_f = angle of elevation
- a_h = constant proportional to probable error of height finder
- a_d = constant proportional to probable error of director.

in the height finder, however, in general the time of flight will not be correctly given. Revision sheets for Report 127 are being prepared. In these it will be shown that the error in the time of flight, Δt , due to errors in the height finder is approximately

$$a_h S R \frac{\Delta t}{aR}.$$

If allowance is made for this error, Δt , in the time of flight it may be shown that the probability that the burst lies within the box of dimensions a , b , c is

$$\frac{.0195 a b c}{a_h a_d^2 S R^3 \omega_\epsilon \omega_\alpha \cos \epsilon_f \left[t^2 + \left(\frac{a_h}{a_d} S R \frac{\Delta t}{aR} \right)^2 \right]} \quad (1a)$$

Since the probability should be independent of the shape of the volume we represent it by

$$f dV$$

in which f is the cofactor of $a b c$ in (1a) and dV is the elementary volume in the neighborhood of the airplane or the projected area.

We now consider the elementary volume, dV , to be in the neighborhood of the burst and state that the probability that the small area A lies within dV is $f dV$. This seems to follow from principles of relativity. The probability (1a) is simply the probability that certain components of the distance between burst and airplane lie within certain limits. It is immaterial whether we consider the airplane fixed and the burst movable or vice versa.

We represent the elementary volume dV by $\Omega r^2 dr$ in which Ω is the solid angle of the side spray. Hence the probability that the target lies within the solid angle of the side spray at a distance between r and $r + dr$ is

$$f \Omega r^2 dr. \quad (1b)$$

Consider a target of projected area A contained within the solid angle Ω at a distance r from the burst.

We proceed to compute the probability that such a target will be hit by at least one effective fragment.

If N is the number of effective fragments from one shell within the side spray of solid angle, Ω , at a distance r from the burst, the number of effective hits per sq. yd. at a distance r will be $\frac{N}{\Omega r^2}$ for a single shot.

Since the number of hits per sq. yd. is $\frac{N}{\Omega r^2}$ it is assumed that the probability of hitting a small element of the area, dA , is $\frac{NdA}{\Omega r^2}$. Hence the probability that dA will not be hit is $(1 - \frac{NdA}{\Omega r^2})$. The total area A is assumed to

be equal to $n dA$ where n is a large number. The probability that none of the elementary areas dA will be hit is

$$\left(1 - \frac{NdA}{\Omega r^2}\right)^n = 1 - \frac{NndA}{\Omega r^2} + \frac{N^2 n(n-1)dA^2}{2(\Omega r^2)^2}, \quad (2)$$

by the binomial theorem. Since n is a large number $(n-1)$ does not differ from n appreciably. Also $ndA = A$. Hence we may write (2) as

$$\left(1 - \frac{NdA}{\Omega r^2}\right)^n = 1 - \frac{NA}{\Omega r^2} + \frac{1}{2!} \left(\frac{NA}{\Omega r^2}\right)^2 - \dots = e^{-\frac{NA}{\Omega r^2}}.$$

This is the probability that A will not be hit; the probability that it will be hit at least once is

$$\left(1 - e^{-\frac{NA}{\Omega r^2}}\right).$$

We now assume that a very large number of shots M are fired. Since the probability that the target will be in the volume $\Omega r^2 dr$ is $\frac{1}{\Omega r^2} dr$, the number of times in which the target will be in this volume is

$fM\Omega r^2 dr$. Since the probability of at least one hit is

$$(1 - e^{\frac{-NA}{\Omega r^2}})$$

for a single shot, the number of times the target will be hit at least once when it lies within the volume

$$\Omega r^2 dr \text{ is } fM\Omega (1 - e^{\frac{-NA}{\Omega r^2}}) r^2 dr.$$

The number of times the target will be hit at least once for all positions of the target will be

$$fM\Omega \int_0^{\infty} (1 - e^{\frac{-NA}{\Omega r^2}}) r^2 dr * \quad (3)$$

The probability of at least one hit is defined as the ratio of the number of times the target is hit at least once to the total number of shots. Hence the probability, p_A , of at least one hit is given by

$$p_A = \frac{fM\Omega \int_0^{\infty} (1 - e^{\frac{-NA}{\Omega r^2}}) r^2 dr}{M} = f\Omega \int_0^{\infty} (1 - e^{\frac{-NA}{\Omega r^2}}) r^2 dr. \quad (4)$$

If N is known as a function of r the integral of (4) can be evaluated numerically. When more information is available of the penetrating power of fragments of various A.A. projectiles as dependent on distance the integrals in (4) will be determined. For the present to obtain an approximate result, we assume that all the fragments N are effective within a range r_e but are ineffective for $r > r_e$. Thus r_e is the effective range of the fragments. For $r > r_e$, $N = 0$ and for $r < r_e$, $N = \text{Const}$. The exponent of e in (4) is therefore zero for $r > r_e$ and the integral is therefore zero for $r > r_e$. Hence

* There is a certain error in this result because expression (1a) holds only if the target is near the point of burst. However, the range of the fragments is comparatively small, and N will be zero for large values of r . Hence expression (3) is approximately correct.

$$p_A = f \Omega \int_0^{r_e} \left(1 - e^{\frac{-NA}{\Omega r^2}}\right) r^2 dr. \quad (4a)$$

In the Appendix the integral of (4a) is evaluated with the result that

$$p_A = fNAr_e \left[1 - \frac{2\sqrt{\pi}}{3} \sqrt{\frac{NA}{\Omega r_e^2}} + \frac{NA}{2\Omega r_e^2} - \dots \right] \quad (4b)$$

We now assume that $\frac{NA}{\Omega r_e^2} \ll 1$; i.e., the average number of

fragments hitting a target A placed at the extreme effective range of the fragments, r_e , is small compared with unity.

If the area A is 1 sq. yd. or less, this condition should be fulfilled in practice. On this assumption we obtain

$$p_A = fANr_e.$$

With a given f and A , p_A varies as Nr_e . The probability p_A is a measure of the effectiveness of a shell in attacking the target A. From this definition it follows that the effectiveness of a shell in attacking a small target, A, is proportional to Nr_e and the effectiveness per lb. is proportional to $\frac{Nr_e}{m}$ where m is the mass of the shell.

Probability of hitting vulnerable parts of the Fuselage or the wings

As was remarked on page 2 the dimensions of an airplane will usually be large compared with the dimensions of the effective side spray. Hence in considering the probability of hitting parts of the fuselage or the wings we should not consider the airplane as a point. For purposes of computation we consider the airplane as made up of two cylinders at right angles to each other, one cylinder representing the fuselage and the other the wings (see figure 1).

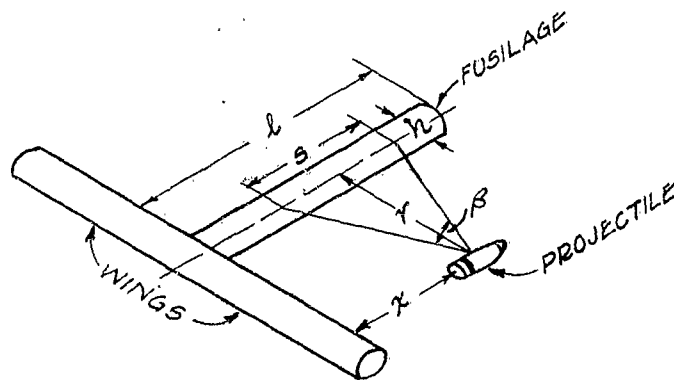


Fig. 1

We assume a projectile with its axis parallel to the axis of the fuselage. The distance of the projectile from the fuselage is denoted by r and its distance from the wings by x . The volume of a ring surrounding the fuselage of inner and outer radii r and $r+dr$ respectively and thickness dx is approximately

$$2\pi r \, dr \, dx.$$

It follows from the argument of page 4 that the probability that the point of burst will lie within the ring is $f \, 2\pi r \, dr \, dx$, and if a very large number of shots, M , are fired, the number bursting inside the given ring will be

$$f \, M \, 2\pi r \, dr \, dx.$$

The side spray is supposed to be bounded by two cones, the difference between the semi-vertex angles of which is designated by β . For brevity, β may be called the angle of opening of the side-spray. It is evident with a given position of the burst the length of fuselage, s , over

which the fragments will be distributed will be approximately βr . If the diameter of the fuselage is h the probability that a fragment will hit the fuselage within an elementary distance ds is $\frac{N\beta h}{r^2} ds$. By the reasoning used used to establish equation (4) it then follows that the probability that the fuselage will receive at least one hit for a given burst is

$$1 - e^{-\frac{N\beta h}{\Omega r^2}} = 1 - e^{-\frac{N\beta r h}{\Omega r^2}} = 1 - e^{-\frac{N\beta h}{\Omega r}}$$

Thus the number of times the fuselage will receive at least one hit from the shell which burst in the ring of volume $2\pi r dr dx$ is

$$2\pi f M (1 - e^{-\frac{N\beta h}{\Omega r}}) r dr dx.$$

The number of times the fuselage will be hit at least once from bursts in all of the rings surrounding it will therefore be

$$2\pi f M \int_0^{\ell} \int_0^{\infty} (1 - e^{-\frac{N\beta h}{\Omega r}}) r dr dx =$$

$$2\pi f M \ell \int_0^{\infty} (1 - e^{-\frac{N\beta h}{\Omega r}}) r dr$$

if ℓ is the length of the fuselage. Dividing the number of times the fuselage is hit by the total number of shots, M , the probability of at least one hit on the fuselage, p_f , is given by

$$p_f = 2\pi f \ell \int_0^{\infty} (1 - e^{-\frac{N\beta h}{\Omega r}}) r dr. \quad (5)$$

If it is assumed that the number of effective fragments is constant up to r_e and zero for $r > r_e$ we have

$$P_f = 2\pi f \ell \int_0^{r_e} \left(1 - e^{\frac{-N\ell h}{Qr}}\right) r \, dr. \quad (5a)$$

It is shown in the appendix that if

$\frac{N\ell h}{Qr_e}$ is small compared with unity, as will

frequently be the case,

$$P_f = f N \ell h r_e. \quad (5b)$$

As before, it is found that the probability of at least one hit is proportional to $N r_e$ and that hence the efficiency per lb. will be proportional to $\frac{N r_e}{m}$.

It should be noted that the axis of the shell is parallel to the axis of the fuselage. If the axis of the shell were parallel to the wings, the probability of at least one hit, would be much smaller. Equations (4) (4a) (4b) (5) and (5b) are based also on the implicit assumption that the effective range of the fragments is considerably less than the probable errors of the coordinates of the predicted position as dependent upon the errors of the height finder and director. If such is not the case, equations (4) and (5) etc. will exaggerate the probability of at least one hit.

For the case where the axis of the projectile is parallel to the wings, the expression for the probability P_w of hitting the wings has the same form as the expressions given above for the probability of hitting the fuselage when the axis of the shell is parallel to the fuselage.

Variation of $\frac{P_A}{m}$, $\frac{P_f}{m}$ and $\frac{P_w}{m}$ with the Caliber

The value of N in equations (4b) and (5b) depends upon the nature of the target, since N is by definition the number of fragments at the given distance, r , capable of penetrating or perforating the target. For the human target (unless protected by armor) and the gas tanks fairly small fragments

should be effective at short ranges. For the heavier parts upon which the structural strengths of the wings and fuselage depend only much larger fragments will be effective. In view of the fact that the personnel are protected to some extent by the wall or sheathing of the fuselage and the gas tanks by the sheathing of the wing, it is reasonable to assume (in the absence of precise experimental data) that only fragments large enough to be caught on the No. 3 screen (opening of dimensions .23"x .23") will be effective.

Data concerning the number of fragments, N, large enough to be caught on the No. 3 screen for the 75 mm shell T-3, the 105 mm shell MI and the 155 mm shell TLEI are given below.*

Shell	N	Average Wt. of fragment lbs.	Wt. of Shell lbs.
75 mm	533	.023	14.7
105 mm	980	.027	33
155 mm †	1340	.056	95

If it is assumed that the average effective range, r_e , for the 75 mm shell is 50 yds.** and that the range is proportional to the average weight of the fragments, we obtain for Nr_e and $\frac{Nr_e}{m}$ for the three shells, the following:

Shell	Nr_e yd.	$\frac{Nr_e}{m}$
75 mm	26,700	1,820
105 mm	57,500	1,750
155 mm	164,000	1,720

It appears from the above that as the caliber increases from 75 mm to 155 mm the efficiency per lb. remains nearly constant.

That this result should be obtained on the assumption that r_e is proportional to the average weight of the fragments follows approximately from elementary considerations.

* Data on the 75 mm shell T-3 obtained from Report No. 126 by N. A. Tolch. The report on the 105 mm shell MI is not yet completed.

** These are the assumed ranges at normal air density. The range should be inversely proportional to the density.

† Data on the 155 mm shell were kindly provided by Picatinny Arsenal.

Let

m = mass of loaded shell

m_e = mass of empty shell

m_3 = mass of fragments too small to be caught on screen No. 3

The product Nr_e is proportional to $m_e - m_3$ since N is the number of fragments large enough to be caught on screen #3 and r_e by hypothesis is proportional to the average weight of the N fragments, i.e., to $\frac{m_e - m_3}{N}$. We may write then

$$Nr_e = k(m_e - m_3)$$

in which k is a factor of proportionality. If the ratio, bursting charge/mass of shell, is a constant, e.g., a , then

$$\frac{Nr_e}{m} = k\left(\frac{m_e}{m} - \frac{m_3}{m}\right) = k\left(1 - a - \frac{m_3}{m}\right).$$

Obviously as the caliber and the mass of the shell increase, the ratio $\frac{m_3}{m}$ should decrease, i.e., the proportion of the shell consisting of small fragments should decrease.

Hence $\frac{Nr_e}{m}$ should increase with the caliber. That it appears to decrease slightly is due to the fact that 'a' increases somewhat with the caliber in shell of current design. This could be offset by reducing slightly the bursting charge of the larger shell. Such a reduction, however, would reduce the initial velocity of the fragments. In deducing the results given above, it was tacitly assumed that the initial velocity of the fragments is the same irrespective of caliber; actually the initial velocity of the fragments of a large caliber shell should be greater than those of a shell of smaller caliber. This increase in velocity with caliber should more than offset the slight reduction of $\frac{Nr_e}{m}$ with caliber as given in the table on page 11.

As mentioned in the foregoing to attack the heavier parts upon which the structural strength of the wings and fuselage depends, it appears likely that much larger shell

than 105 mm would be required but how large is not definitely known. Furthermore, if the pilot were given armor protection, larger fragments even than those produced by a 105 mm shell would probably be required to penetrate it.

Numerical results for p_A for various conditions.

We calculate the probability that the pilot with $A=1$ will be hit by a fragment for various conditions using equation (4b) of which the f is given by the co-factor of $a b c$ in (1a) under various conditions with $a_h = 3.1 \times 10^{-6}$, $a_d = .1$ as follows:

Altitude yd.	Present Slant range yd.	Air Speed yd/sec	Course	Pro- jectile	Time of Flight	N**	r_o^\dagger	p_A
10,000	18,250	150	30° coming	3" Mk IX	30*	429	134	$\frac{1}{49,000}$
10,000	18,250	150	30° "	105 mm MI	30	980	170	$\frac{1}{17,000}$
5,000	9,100	50	30° "	3" Mk IX	15	429	80	$\frac{1}{280}$
5,000	9,100	50	30° "	105 mm MI	15	980	100	$\frac{1}{100}$

Probability of at least one hit when the errors of the fire control apparatus are small

In the deduction of equations (4b) and (5b) it was tacitly assumed that the probable errors of the coordinates of the predicted future position of the airplane are large compared with the effective range of the fragments. If the shell are not larger than 105 mm and the errors in the predicted future position are as great as those obtained with existing fire control equipment at altitudes greater than say 10,000 ft., this assumption is probably correct. If the fire control apparatus is improved or the shell are larger, or both, the assumption may no longer be correct. To take account of the reduction in the probability that a

* This would require a very great muzzle velocity.

** Assuming the fragments, caught on the #3 screen are effective.

† These ranges make allowance for the reduced air density. See footnote on page 11.

hit will occur in a given volume dV as the distance r from the airplane increases, we take approximately in place of (1b)

$$f \Omega e^{\frac{-r^2}{a^2}} r^2 dr. \quad (6)$$

In this .477a is a sort of mean of the probable errors of the three coordinates of the predicted future position. By the aid of (6) we obtain in place of (4)

$$P_A = f \Omega \int_0^{\infty} (1 - e^{\frac{-NA}{r^2}}) e^{\frac{-r^2}{a^2}} r^2 dr, \quad (7)$$

and in place of (4a)

$$P_A = f \Omega \int_0^{r_e} (1 - e^{\frac{-NA}{r^2}}) e^{\frac{-r^2}{a^2}} r^2 dr. \quad (7a)$$

We shall not attempt to evaluate (7a) in general terms but merely point out that in the limit as the mean probable error .477a approaches 0, the target will certainly be hit if any part of it is included on the solid angle of the side spray. In other words, if the precision of the fire control apparatus and the ammunition are sufficiently high, the probability of at least one hit is independent of the fragmentation characteristics of the projectile. However, great improvement will have to be made in the fire control apparatus before such a condition arises.

Although with the present fire control apparatus, the probability of a hit will not be independent of the fragmentation characteristics of the projectile, nevertheless as the caliber increases the assumption that the effective range of the fragments is always small compared with the probable errors of the coordinates of the predicted position will cease to hold. It is estimated that at a slant range of 15,000 yds., an altitude of 10,000 yds. and an airplane

speed of 300 miles/hour about 4% of the fragments of the 105 mm shell MI will have ranges exceeding the mean probable error of the coordinates of the predicted position; as the caliber increases the proportion of such fragments will increase and for such shell equation (4a) will overestimate the probability of hitting. Hence when the caliber reaches a certain magnitude the efficiency per lb. in attacking personnel will begin to diminish, although the efficiency per lb. in attacking the structure of the airplane will continue to increase until a larger caliber is reached. Because of lack of the fundamental data, an accurate estimate cannot be made of the caliber at which the efficiency per lb. for attacking personnel and structure respectively begins to fall off. It is probable that for attacking personnel no appreciable loss will occur with the present fire control at high altitudes and a 300 mi/hr. air speed until the caliber exceeds 4.7" while for attacking the structure, the efficiency per lb. will continue to increase until the caliber reaches 8".

The probable error in the predicted position increases with the slant range, the speed of the airplane and time of flight and decreases as the precision of the fire control instruments increases. It follows that for great slant ranges, long times of flight and high air speeds the optimum caliber is larger than for shorter ranges and times of flight and lower air speeds. The optimum caliber for a given slant range, time of flight and air speed diminishes as the precision of the fire control increases.

With the present fire control instruments in attacking an airplane having a high speed, flying at high altitudes, the efficiency per lb. for attacking the structure of the airplane probably continues to increase until an 8" caliber is reached. It is not at all certain, however, in view of the results available, that even an 8" shell would be able by its fragmentation to cause a wing to fail or the fuselage to break in two. In view of this uncertainty it would seem preferable to depend upon attacking the personnel, the gas tanks etc. by shell of relatively small caliber, 4.7" or less depending upon the fire control apparatus, the altitude etc. On the other hand if a relatively light armor protection were provided for the pilot, bomber and vital parts like the bomb sight, fuel lines, etc., the effectiveness of the 4.7" or smaller shell would be very much reduced and to make the airplane vulnerable, larger and larger shell would have to be used as the amount of armor protection is increased.

Probability of a direct hit on an airplane flying overhead

In view of the relative ineffectiveness of the fragments of a shell against airplanes, it is interesting to compute the probability of a direct hit. If a shell were armed with a supersensitive fuze with a delay of about .0005, the explosion would occur inside the structure of the wing or fuselage and would in all probability disable the airplane immediately.

Consider an airplane flying at a constant altitude, y, as shown in figure 2 on page 17 with the future position of the airplane in the X Y plane. Let the probable errors in the x and z coordinates of the predicted future position be Δx and Δz . It is apparent from the figure that

$$\Delta x = \frac{R \Delta \epsilon_f}{\sin \epsilon_f}, \Delta z = R \Delta \theta = R \Delta \alpha_f \cos \epsilon_f,$$

using the notation of Report 127. From the results given in Hayes' 'Elements of Ordnance' p. 481, it is apparent that the probability p_d of a direct hit on an airplane of plan area, Q, is

$$p_d = \frac{.477^2 Q}{\pi \Delta x \Delta z} = \frac{.072 Q}{\Delta x \Delta y} = \frac{.072 Q \sin^2 \epsilon_f}{R^2 \Delta \epsilon_f \Delta \alpha_f \cos \epsilon_f}$$

provided there are no systematic errors in the fire control apparatus. As shown on page 12 of report 127 (Addendum).

$$\Delta \epsilon_f \Delta \alpha_f = a^2 \omega_\epsilon \omega_\alpha \left[t^2 + \left(\frac{a_h}{a_d} SR \frac{\partial t}{\partial R} \right)^2 \right]$$

In view of this, we obtain

$$p_d = \frac{.072 Q \tan^2 \epsilon_f}{a_d^2 R^2 \omega_\epsilon \omega_\alpha \left[t^2 + \left(\frac{a_h}{a_d} SR \frac{\partial t}{\partial R} \right)^2 \right]} \quad (8a)$$

If A represents the projected area of the pilot we obtain p_A the probability of hitting the pilot by a fragment from (4b) on substituting for f its value as given by (1a)

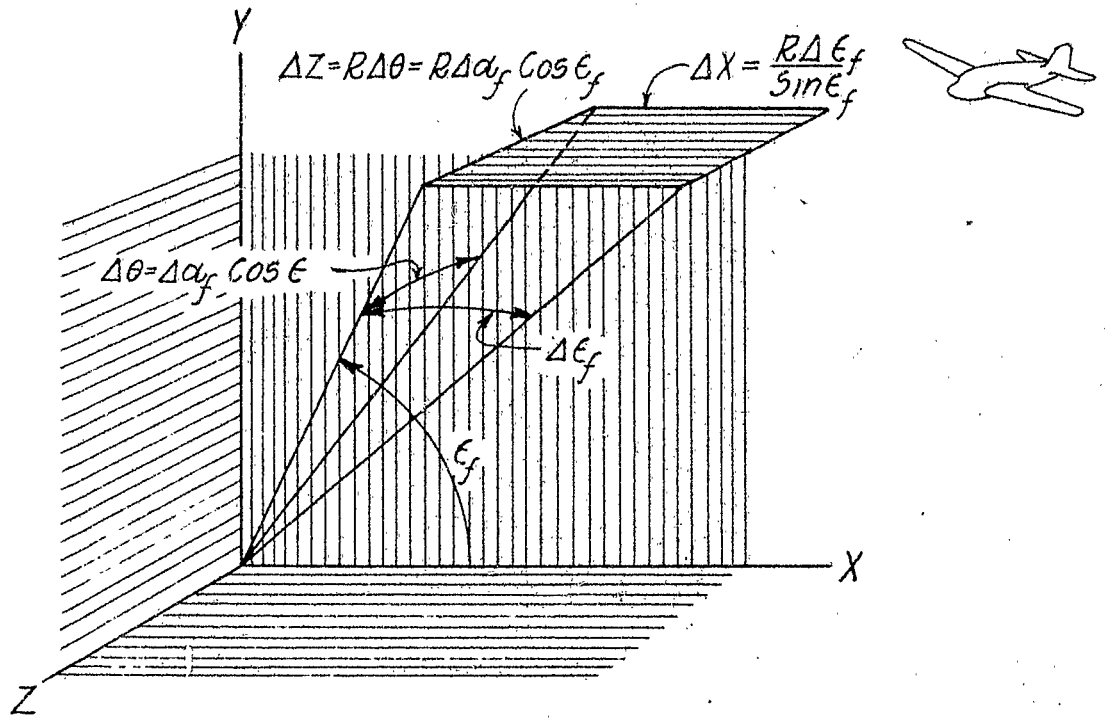


Fig. 2

$$P_A = \frac{.0195 \text{ NA } r_e}{a_h a_d^2 \text{ SR}^3 \omega_\epsilon \omega_\alpha \cos \epsilon_f \left[t^2 + \left(\frac{a_h}{a_d} \text{ SR } \frac{at}{aR} \right)^2 \right]}$$

We obtain for the ratio, $\frac{P_d}{P_A}$,

$$\frac{P_d}{P_A} = \frac{.072}{.0195} \frac{a_h \text{ SR } Q \sin \epsilon_f}{\text{NA } r_e} = \frac{3.7 a_h \text{ SR } Q \sin \epsilon_f}{\text{NA } r_e} \quad (9)$$

Suppose the airplane is flying nearly overhead at an altitude of 10,000 yds. then $S=R=10,000$ and $\sin \epsilon_f = 1$.

From page 6 of Report 127, we take $a_h = 3.1 \times 10^{-6}/\text{yd}$.

For a modern bomber we take Q the plan area as 150 yd^2 . For the 105 mm shell, counting the fragments caught on the number 3 screen, we have $N = 980$. We assume that $r_e = 170 \text{ yd}$. (low air density). Upon substituting in (9) we obtain

$$\frac{P_d}{P_A} = \frac{3.7 \times 3.1 \times 10^{-6} \times 10^8 \times 150}{980 \times 170} = 1106^3$$

If the altitude were reduced to 5000 yd. other conditions being equal we should obtain:

$$\frac{P_d}{P_A} = .44^{26}$$

This comparison seems to neglect the probability of hitting other parts of the airplane besides the pilot. It appears, however, that for attack by fragments, the pilot is the most vulnerable part of the airplane. While

the fragments may puncture the gas tanks, such punctures should not cause the bombing mission to be abandoned immediately although they might cause an eventual forced landing. The probability of the fragments cutting the control cables or damaging the engine seriously is small and, as remarked, the fragments of shell of a 105 mm caliber are incapable of materially damaging the structural framework of the airplane.

The probability of hitting the pilot p_A was calculated on the assumption that the pilot is not protected by armor. If he is protected the probability p_A would be very much reduced. Furthermore the probability of a direct hit, p_d , would be just as great with a smaller shell, e.g., a 3" one, if the time of flight were equal, as by a 105 mm shell. On the other hand if great improvements are made in the height finder, a great increase in the probability of hitting the pilot by fragments would ensue. It appears from the foregoing that to attack airplanes flying at high altitudes it would be much better with the present height finder to employ high velocity shell of small caliber armed with supersensitive fuzes than to employ shell of much larger caliber armed with time fuzes. There seems to be little doubt that an explosion of a shell as small as one of a 3" caliber within the structure of the wings or fuselage would immediately disable the airplane. On the other hand unless great improvements are made in both the director and height finder, attempts to attack an airplane flying at an altitude of 10,000 yds. and an air speed of 300 mi/hr by shell armed either with time or percussion fuzes will have little chance of success (see table on p. 13).

Resume

It is pointed out that there is no authoritative basis for the widely held belief that a 'hit' will necessarily disable an airplane. There is a need for the development of a method for estimating the probability of disabling an airplane.

The probability of hitting a small projected area, A , and the probability of hitting the fuselage for a favorable orientation of the axis of the shell are computed. It is shown that both these probabilities are approximately proportional to Nr_e in which N is the number and r_e the average effective range of the effective fragments. On certain more or less plausible assumptions, it is shown that even in attacking unprotected personnel a 105 mm shell should be as efficient per lb. as a 75 mm one.

The conclusion that the efficiency per lb. is practically independent of the caliber is based on the assumption that the probable error of the predicted position is greater than the effective range of the fragments. As the precision of fire increases this condition no longer holds. It follows that for attacking personnel and vulnerable parts of the airplane by fragments the optimum caliber should increase as the range, time of flight and air speed of the airplane increase and decrease as the precision of the fire control increases. The caliber of the shell should of course increase as the armor protection increases.

The probability of a direct hit is computed. It is shown that with the present height finders the probability of a direct hit at high altitudes e.g., 10,000 yd. is about as great as the probability of hitting the pilot by a fragment of a 105 mm shell for a given time of flight. In view of this result, it is suggested that to attack airplanes at high altitudes especially if they provide armor protection for the pilot, shell of relatively small caliber armed with supersensitive fuzes should be employed.

Recommendations

As pointed out on page 6, to evaluate accurately the probability of hitting a small target, it is necessary to know N the number of effective fragments as a function of r the distance from the burst. It is recommended that a program be initiated to determine the function $N(r)$ for projectiles of various calibers and types acting against dummies variously clothed and protected by armor, and against structures of the sorts to be found on airplanes.

In view of the fact that with the present fire control apparatus the probability of a direct hit at high altitudes appears to be as great as the probability of hitting the unprotected pilot by a fragment of a 105 mm shell, it is recommended that serious consideration be given to the advisability of the development of an antiaircraft artillery firing shell armed with supersensitive fuzes having a delay of about .0005 sec. While such an artillery would probably be less effective on the whole than the existing type against unarmored airplanes; against airplanes providing armor protection for the pilot and other vital parts, the shell armed with the impact fuzes should be more effective than those armed with time fuzes.

R. H. Kent

Appendix

Evaluation of Integrals

A. Evaluation of $\int_0^{r_e} (1 - e^{-\frac{NA}{\Omega r^2}}) r^2 dr$ $\frac{NA}{\Omega r_e^2} < 1$

Let $k = \frac{NA}{\Omega}$
and $r = \frac{1}{\sqrt{z}}$

Then $\int_0^{r_e} (1 - e^{-\frac{k}{r^2}}) r^2 dr = \frac{r_e^3}{3} + \frac{1}{2} \int_{\infty}^{\frac{1}{r_e^2}} e^{-kz} z^{-5/2} dz$

Integrating by parts successively,

$$\begin{aligned} \frac{1}{2} \int_{\infty}^{\frac{1}{r_e^2}} e^{-kz} z^{-5/2} dz &= \frac{-r_e^3 e^{-\frac{k}{r_e^2}}}{3} + \frac{2kr_e e^{-\frac{k}{r_e^2}}}{3} \\ &+ \frac{4k^2 e^{-\frac{k}{r_e^2}}}{3r_e} + \frac{4k^3}{3} \int_{\infty}^{\frac{1}{r_e^2}} z^{1/2} e^{-kz} dz. \end{aligned}$$

$$\int_{\infty}^{\frac{1}{r_e^2}} z^{1/2} e^{-kz} dz = - \int_0^{\infty} z^{1/2} e^{-kz} dz + \int_0^{\frac{1}{r_e^2}} z^{1/2} e^{-kz} dz.$$

$$\text{But } \int_0^{\infty} z^{1/2} e^{-kz} dz = \frac{1}{2k} \sqrt{\frac{\pi}{k}}$$

$$\text{and } \int_0^{r_e} z^{1/2} e^{-kz} dz = \int_0^{r_e} z^{1/2} (1 - kz + \frac{k^2 z^2}{2} - \dots) dz = \frac{2}{3r_e^3} - \frac{2k}{5r_e^5} + \frac{k^2}{7r_e^7} - \dots$$

$$\text{Then } \int_0^{r_e} (1 - e^{-\frac{k}{r_e^2} r^2}) r^2 dr = \frac{r_e^3}{3} - \frac{r_e^3}{3} e^{-\frac{k}{r_e^2}} + \frac{2kr_e}{3} e^{-\frac{k}{r_e^2}} - \frac{4k^2 r_e}{3r_e} e^{-\frac{k}{r_e^2}}$$

$$= \frac{2k^2 \sqrt{\pi}}{3} + \frac{8k^3}{9r_e^3} - \frac{8k^4}{15r_e^5} + \frac{4k^5}{21r_e^7} - \dots$$

$$\text{Substituting } e^{-\frac{k}{r_e^2}} = 1 - \frac{k}{r_e^2} + \frac{k^2}{2r_e^4} - \frac{k^3}{6r_e^6} + \frac{k^4}{24r_e^8} - \frac{k^5}{120r_e^{10}} + \dots$$

$$\int_0^{r_e} (1 - e^{-\frac{k}{r_e^2} r^2}) r^2 dr = kr_e - \frac{2k^2 \sqrt{\pi}}{3} + \frac{k^2}{2r_e} - \frac{k^3}{18r_e^3} + \frac{k^4}{120r_e^5} - \frac{k^5}{840r_e^7} + \dots$$

$$f \Omega \int_0^{r_e} (1 - e^{-\frac{NA}{\Omega r_e^2} r^2}) r^2 dr = fNA r_e \left[1 - \frac{2\sqrt{\pi}}{3} \sqrt{\frac{NA}{\Omega r_e^2}} + \frac{NA}{2\Omega r_e^2} - \frac{(NA)^2}{18\Omega^2 r_e^4} + \frac{(NA)^3}{120\Omega^3 r_e^6} - \dots \right]$$

B. Evaluation of $2\pi f \int_0^{r_e} (1 - e^{\frac{-N\beta h}{\Omega r}}) r dr$ $\Omega = 2\pi\beta$
and
 $\frac{N\beta h}{\Omega r_e} < 1$

Let $a = \frac{N\beta h}{\Omega}$

and $r = \frac{1}{y}$

Then $\int_0^{r_e} (1 - e^{\frac{-N\beta h}{\Omega r}}) r dr = \int_{\infty}^{\frac{1}{r_e}} -(1 - e^{-ay}) \frac{dy}{y^3} = \frac{r_e^2}{2} + \int_{\infty}^{\frac{1}{r_e}} \frac{e^{-ay}}{y^3} dy$

Integrating by parts successively,

$$\int_{\infty}^{\frac{1}{r_e}} \frac{e^{-ay}}{y^3} dy = -\frac{e^{-\frac{a}{r_e}}}{2} \frac{r_e^2}{e} + \frac{a e^{-\frac{a}{r_e}}}{2} r_e - \frac{a^2}{2} \int_{\frac{1}{r_e}}^{\infty} \frac{e^{-ay}}{y} dy$$

The integral, $\int_{\frac{1}{r_e}}^{\infty} \frac{e^{-ay}}{y} dy$, can be developed into

a convergent series with a remainder integral, which is expressed as a divergent but summable series.

Let $ay = t$

Then $\int_{\frac{1}{r_e}}^{\infty} \frac{e^{-ay}}{y} dy = \int_{\frac{a}{r_e}}^{\infty} \frac{e^{-t}}{t} dt$

$$= \overset{\text{I}}{\int_1^{\infty} \frac{e^{-t}}{t} dt} - \overset{\text{II}}{\int_0^1 \frac{(1-e^{-t})}{t} dt} + \overset{\text{III}}{\int_0^{\frac{a}{r_e}} \frac{(1-e^{-t})}{t} dt} - \overset{\text{IV}}{\int_1^{\frac{a}{r_e}} \frac{dt}{t}}$$

I + II = $-\gamma$ (divergent but summable)

IV = constant

III = absolutely and uniformly convergent series.

$$\gamma = \int_0^1 \frac{(1-e^{-t})}{t} dt - \int_1^{\infty} \frac{e^{-t}}{t} dt$$

$$\gamma = \lim_{m \rightarrow \infty} \left[\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{m} - \log m \right] = 0.5772157$$

(Euler's constant)

$$\int_1^{\frac{a}{r_e}} \frac{dt}{t} = \log \frac{a}{r_e}$$

$$\int_0^{\frac{a}{r_e}} \frac{(1-e^{-t})}{t} dt = \int_0^{\frac{a}{r_e}} \left(1 - \frac{t}{2} + \frac{t^2}{3!} - \frac{t^3}{4!} + \dots \right) dt$$

$$= \frac{a}{r_e} - \frac{a^2}{2!2!r_e^2} + \frac{a^3}{3!3!r_e^3} - \frac{a^4}{4!4!r_e^4} + \dots$$

$$\int_{\frac{1}{r_e}}^{\infty} \frac{e^{-ay}}{y} dy = -0.5772157 - \log \frac{a}{r_e} + \frac{a}{r_e} - \frac{a^2}{4r_e^2} + \frac{a^3}{18r_e^3} - \frac{a^4}{96r_e^4} + \dots$$

$$\int_0^{r_e} (1 - e^{-\frac{a}{r}}) r dr = \frac{r_e^2}{2} - \frac{e^{-\frac{a}{r_e}} r_e^2}{2} + \frac{a e^{-\frac{a}{r_e}} r_e}{2} - \frac{a^2}{2} \left[-0.5772157 \right.$$

$$\left. -\log \frac{a}{r_e} + \frac{a}{r_e} - \frac{a^2}{4r_e^2} + \frac{a^3}{18r_e^3} - \frac{a^4}{96r_e^4} + \dots \right]$$

$$\text{Substituting } e^{-\frac{a}{r_e}} = 1 - \frac{a}{r_e} + \frac{a^2}{2r_e^2} - \frac{a^3}{6r_e^3} + \frac{a^4}{24r_e^4} - \frac{a^5}{120r_e^5} + \dots,$$

$$\int_0^{r_e} (1 - e^{-\frac{a}{r}}) r dr = ar_e + \frac{a^2}{2} (0.5772157 - \frac{3}{2} + \log \frac{a}{r_e}) - \frac{a^3}{6r_e}$$

$$+ \frac{a^4}{48r_e^2} - \frac{a^5}{360r_e^3} + \dots$$

$$2\pi f l \int_0^{r_e} (1 - e^{-\frac{N\phi h}{2\pi r}}) r dr =$$

$$f l N h r_e \left[1 + \frac{N h}{4\pi r_e} (\log \frac{N h}{2\pi r_e} - 0.9227843) \right.$$

$$\left. - \frac{(N h)^2}{24\pi^2 r_e^2} + \frac{(N h)^3}{384\pi^3 r_e^3} - \frac{(N h)^4}{5760\pi^4 r_e^4} + \dots \right]$$

B. I. Hart

B. I. Hart

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